A method for analyzing the dynamic response of a structural system with variable mass, damping and stiffness

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In this paper, a method for analyzing the dynamic response of a structural system with variable mass, damping and stiffness is first presented. The dynamic equations of the structural system with variable mass and stiffness are derived according to the whole working process of a bridge bucket unloader. At the end of the paper, an engineering numerical example is given.

Keywords: Dynamic response, variable mass, variable damping, variable stiffness

1. Introduction

The bridge bucket unloaders and container cranes are now the main facilities using for loading and unloading the goods in ports around the world. The high working efficiency requires bigger and faster cranes. In analyzing bridge bucket unloaders, conventional structural analysis methods produce some errors, sometimes causing serious problems because of lack of consideration of the complex dynamic behavior.

The bridge crane is a machine with repeating movements including accelerating, hoisting, traversing and decelerating. There are also two combined movements such as hoisting with traversing and lowing with traversing. As the crane becomes heavier, the influence of vibration caused by start lifting, load moving and unloading will become significant and must be considered in the crane structure analysis. Based on above and our experience, we know that the crane steel structure bears the complex and strenuous vibrations during its operation. During the trolley and lifting load moving, the mass, damping and stiffness matrices of the crane structure will change simultaneously. So the complicated dynamic response should be taken into account in the actual structural design. Using the finite element method and the wide application of the computer, research and analysis on the vibration of the crane steel structure was conducted. However, the work including the dynamic response of the crane with the consideration of the aforementioned factors is quite rare. In this paper, the variable mass, damping and stiffness dynamic equations and the solution are presented with sufficient consideration of the above factors' influence, which comparatively really simulates and analyzes all these kinds of responses of the whole dynamic process during the unloader's actual operations.

2. Duty cycle description

One duty cycle for bridge bucket unloader consists of bucket loading goods from ship, lifting, trolley traveling back to the dock, unloading the goods, trolley traveling forth to the ship and bucket lowing for loading. There are six basic motions (i.e. bucket closing for loading, hoisting, trolley traveling back with loads, bucket opening for unloading, trolley traveling forth with empty bucket, bucket lowing) and their combinations, shown as Fig. 1. Based on the actual work process, The forces in the crane structure are relatively small for the three motions of bucket closing, trolley traveling with empty bucket and bucket lowing. So in this paper, we only simulate the rest three motions of hoisting with loads, trolley traveling back with loads

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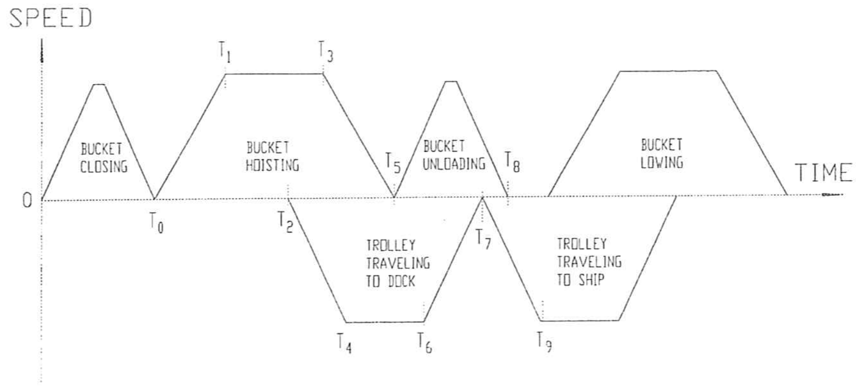

Fig. 1. Duty cycle of unloader.

and unloading. See the interval $T_0 \sim T_8$ in Fig. 1 for reference.

Based on Fig. 1, there are seven moving combinations in the interval $T_0 \sim T_8$ described as follows:

1) Hoisting Accelerating phase: The lifting weight leaves the ground and continues to be accelerated at the acceleration of $\ddot{V}_{ow}$. The system stiffness matrix changes with different elevation of lifting weight at any moment.
2) Hoisting at uniform speed: The lifting weight ascends at the rated hoist speed.
3) Hoisting at uniform velocity and the trolley accelerating back move: When the lifting weight is ascending at the rated hoist speed, in the mean time, the trolley starts to move back (to dock side) at the acceleration of a_x. The system mass matrix changes with the trolley's movement.
4) Hoisting decelerating and the trolley accelerating back move: When the lifting weight is ascending at the deceleration of $\ddot{V}_{ow}$, the trolley continues to move back at the acceleration of a_x.
5) Hoisting decelerating phase and the trolley moving back at the uniform speed: The lifting weight is ascending at the deceleration of $\ddot{V}_{ow}$, the trolley moves back at the rated traversing speed of V_x.
6) Trolley moving back at the uniform speed and opening the bucket for unloading: The trolley continues to move back at uniform speed motion. The bucket's open-close rope begins to open the bucket for unloading at the same time.
7) Trolley decelerating moving back and opening the bucket for unloading: When the trolley moves back at the deceleration of a_x, the bucket continues unloading.

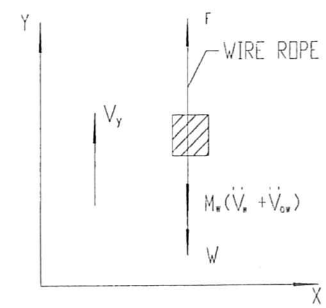

Fig. 2. Hoist diagram.

3. Establishment and solving method of variable mass, damping and stiffness dynamic equations

Based on above description, we know that the hoisting changes the length of hoisting ropes causing the variation of the system stiffness matrix and the movement of the trolley and goods causes the variation of system mass and stiffness matrixes. The dynamic equations are discussed and established in the following subdivisions.

3.1. Vibration equation of the lifting weight

1) Y – direction

Shown in Fig. 2, the wire rope tension is F at a certain time, the mass of the lifting weight is M_w, $W = M_w \cdot g$, the dynamic equation in Y-direction is:

$$M_w(\ddot{V}_w + \ddot{V}_{ow}) + c_w \dot{V}_w + W = F \tag{1}$$

Where V_w is the elastic displacement of the lifting weight's center of gravity and F is the elastic restoring force of the wire rope. C_w is damping.

Assuming the initial vertical length of the wire rope is l_0, l_t is the vertical length at the time t, the elastic extension of the rope is δ_1 and the vertical displacement at the trolley position is V'_w.

$$V_{ow} = l_0 - l_t \tag{2}$$

Where V_{ow} is the shortening of the vertical length of the wire rope only caused by hoisting the lifting weight. From Fig. 3, we can establish the equation:

$$l_t + \delta_l + V_{ow} + V_w = l_0 + V'_w \tag{3}$$

From Eqs (2) and (3), we get:

$$\delta_l = V'_w - V_w \tag{4}$$

The tension F of the wire rope at the time t is given by:

$$F = k_L(t) \cdot \delta_l = k_L(t)(V'_w - V_w) \tag{5}$$

Where $k_L(t)$ is the stiffness of the wire rope at the t-time:

$$k_L(t) = E_r \cdot A_r / l_t \tag{6}$$

Where E_r is the elastic modulus of the wire rope and A_r is the cross – section area of the wire rope.

Replace the F in Eq. (1) by the F in Eq. (5), the dynamic equation in Y – direction is got:

$$\begin{aligned} &M_w \ddot{V}_w + C_w \dot{V}_w + k_L(t) V_w \\ &= k_L(t) V'_w - (W + M_w \cdot \ddot{V}_{ow}) \\ &= k_L(t) V'_w + Q \end{aligned} \tag{7}$$

Where $\ddot{V}_{ow}$ varies with the different moving phases as follows:

$$\ddot{V}_{ow} = \begin{cases} \ddot{V}_{ow} & t \in [T_0, T_1] \\ 0 & t \in [T_0, T_3] \\ -\ddot{V}_{ow} & t \in [T_3, T_5] \\ 0 & t \geqslant T_5 \end{cases} \tag{8}$$

Due to the unloading $T_5 - T_8$, the M_w in Eq. (7) is defined as:

$$\begin{aligned} M_w = M_1 + \left(1 - \frac{t - T_3}{T_8 - T_5}\right) \\ M_2 | T_5 \leqslant t \leqslant T_8 \end{aligned} \tag{9}$$

$$M_w = M_1 | T_5 > t, \text{ or } t > T_8 \tag{9'}$$

Where M_1 is the mass of the bucket, M_2 is the mass of goods.

2) X – direction

The vibration of the lifting weight at the horizontal (X) direction. The lifting weight exerts different horizontal force F_x on the crane structure in different time interval described as follows:

A) In $T_0 - T_2$, the lifting weight has no movement in X – direction.

$$F_x = 0 \tag{10}$$

B) In $T_2 - T_4$, following the horizontal acceleration of the trolley, the lifting weight gradually acquires the same horizontal speed as the trolley, which exerts the horizontal force F_x on the structure. The F_x is:

$$\begin{aligned} F_x = -M_w \cdot a_x \cdot \left(\frac{t - T_2}{T(t)/4}\right) | \\ (t - T_2) \leqslant T(t)/4 \end{aligned} \tag{11}$$

$$F_x = -M_w \cdot a_x | (t - T_2) > T(t)/4 \tag{11'}$$

Where $T(t)$ is the swing period of the lifting weight at the time t, and its swing angle is relatively small ($\theta < 5°$), we can calculate the $T(t)$ using the following formula:

$$T(t) = 2\pi \sqrt{l_t / g} \tag{12}$$

C) In $T_4 - T_6$, The lifting weight moves horizontally at the same speed as the trolley,

$$F_x = 0 \tag{13}$$

D) In $T_6 - T_7$, the trolley has a horizontal decelerated motion, according to the analysis in B):

$$\begin{aligned} F_x = M_w \cdot a_x \cdot \left(\frac{t - T_6}{T(t)4}\right) | (t - T_6) \\ \leqslant T(t)/4 \end{aligned} \tag{14}$$

$$F_x = M_w \cdot a_x | (t - T_6) > T(t)/4 \tag{14'}$$

3.2. Structural dynamic equation

In order to establish the dynamic equation of the structural system, we first study the displacement relationship among the lifting weight's center of gravity, the hanging point on the trolley and the related structure nodes, shown in Fig. 4.

Assuming at a certain moment t, the left and right wheels of the trolley have been respectively on the spaces among the nodes of i, j, k and l of the boom discretized by the finite element. The V'_w in Fig. 4 is same as the V'_w in Fig. 3. According to the Fig. 4, the displacement relationship between the V'_w and the

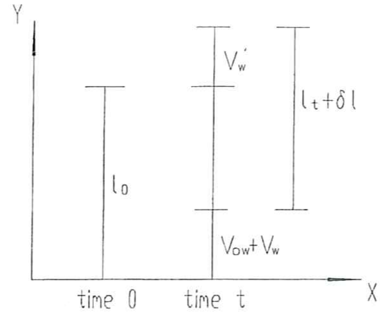

Fig. 3. Wire ropes length change.

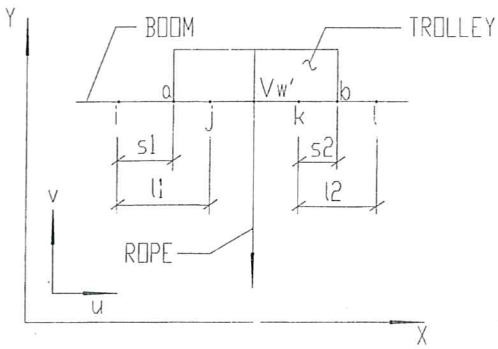

Fig. 4. Trolley move.

structure nodes of i, j, k and l can be obtained,

$$V'_w = 1/2(V_a + V_b) \tag{15}$$

Where V_a is the displacement of the trolley's left wheel, V_b is the displacement of the right wheel.

$$\begin{aligned} V_a &= (1 - s_1/l_1)V_i + s_1/l_1 V_j \\ &= (1 - \alpha_1)V_i + \alpha_1 V_j \end{aligned} \tag{16}$$

$$\begin{aligned} V_b &= (1 - s_2/l_2)V_k + s_2/l_2 V_l \\ &= (1 - \alpha_2)V_k + \alpha_2 V_l \end{aligned} \tag{17}$$

s_1 is the distance from the trolley's left wheel to the node i, shown in Fig. 4.
s_2 is the distance from the trolley's right wheel to the node k, shown in Fig. 4.
l_1 is the distance between node i and node j, shown in Fig. 4.
l_2 is the distance between node k and node l, shown in Fig. 4.
s_1 and s_2 are based on Eqs (18) and (19):

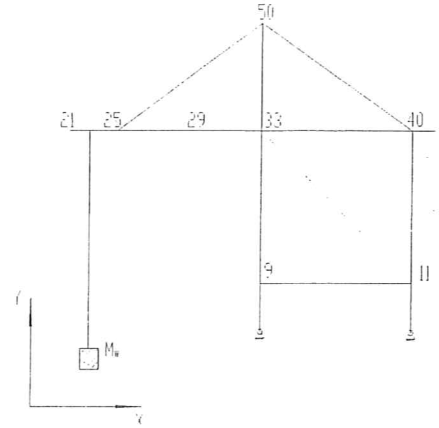

Fig. 5. Elevation of crane.

$$s_1 = \begin{cases} x_0 - x_i & t \in [T_0, T_2] \\ x_0 + 1/2 \cdot a_x(t - T_2)^2 - x_i & t \in [T_2, T_4] \\ x_0 + C_1 + V_x(t - T_4) - x_i & t \in [T_4, T_6] \\ x_0 + C_2 + V_x(t - T_6) & t \in [T_6, T_7] \\ \quad -1/2 \cdot a_x(t - T_6)^2 - x_i & \\ x_0 + C_3 - 1/2 \cdot a_x(t - T_7)^2 & t \in [T_7, T_8] \\ \quad - x_i & \end{cases} \tag{18}$$

Where,

$$\begin{aligned} C_1 &= 1/2 \cdot a_x(T_4 - T_2)^2, \\ C_2 &= C_1 + V_x(T_6 - T_4), \\ C_3 &= C_2 + V_x(T_7 - T_6) \\ &\quad -1/2 \cdot a_x(T_7 - T_6)^2 \\ s_2 &= x_i + s_1 + L_{ab} - x_k \end{aligned} \tag{19}$$

Where, x_0 is the initial x coordinates of the trolley's left wheel, x_i is the x coordinates of node i in Fig. 4. L_{ab} is the distance between the trolley's left wheel a and the right wheel b, shown in Fig. 4.

Substituting Eqs (16) and (17) into Eq. (15), we can obtain:

$$\begin{aligned} V'_w = 1/2\{&(1 - \alpha_1)V_i + \alpha_i V_j \\ &+(1 - \alpha_2)V_k + \alpha_2 V_l\} \end{aligned} \tag{20}$$

Now, we know the displacement relationship between the structure and the hanging point on the trolley.

The structural dynamic equation is:

$$M\ddot{U} + C\acute{U} + KU = f(t) \tag{21}$$

$$U = [U_1, V_1, \ldots, U_i, V_i, \ldots, U_N, V_N]^T \tag{21'}$$

Where N is the total node number of the structure.

Following discusses the effect of the moving trolley and the lifting weight on the M, K and $f(t)$ in the structural dynamic equation.

$$\begin{array}{cc} & \begin{array}{ccccccccccccccc} & \cdots & u_i & v_i & \cdots & u_j & v_j & \cdots & u_k & v_k & \cdots & u_l & v_l & \cdots & \end{array} \\ \Delta M(t) = m & \left[\begin{array}{ccccccccccccccc} 0 \\ & \ddots \\ & & (1-\alpha_1) \\ & & & (1-\alpha_1) \\ & & & & \ddots \\ & & & & & \alpha_1 \\ & & & & & & \alpha_1 \\ & & & & & & & \ddots \\ & & & & & & & & (1-\alpha_2) \\ & & & & & & & & & (1-\alpha_2) \\ & & & & & & & & & & \ddots \\ & & & & & & & & & & & \alpha_2 \\ & & & & & & & & & & & & \alpha_2 \\ & & & & & & & & & & & & & \ddots \\ & & & & & & & & & & & & & & 0 \end{array}\right] \end{array} \tag{30}$$

3.2.1. The effect of the trolley's horizontal movement on M and $f(t)$

(1) For the mass matrix M

When the trolley is moving, its mass is distributed to the nodes of i, j, k and l as follows:

$$\begin{aligned} M_i &= (1-\alpha_1)m \quad M_j = \alpha_1 \cdot m \\ M_k &= (1-\alpha_2)m \quad M_l = \alpha_2 \cdot m \end{aligned} \tag{22}$$

Where, m is the half mass of the trolley. $\alpha = s_1/l_1$ and $\alpha_2 = s_2/l_2$, see Eqs (16) and (17).

(2) For the load $f(t)$

A) X – direction (horizontally)

The trolley gives the structure horizontal forces in the phase of acceleration and deceleration described as follow:

a. In $T_0 - T_2$, there is no horizontal force.
b. In $T_2 - T_4$

$$\begin{aligned} f_{ix}^o &= -(1-\alpha_1)ma_x \quad f_{jx}^o = -\alpha_1 ma_x \\ f_{kx}^o &= -(1-\alpha_2)ma_x \quad f_{lx}^o = -\alpha_2 ma_x \end{aligned} \tag{23}$$

c. In $T_4 - T_6$, there is no horizontal force.
d. In $T_6 - T_7$, the trolley moves at an acceleration of $-a_x$.

$$\begin{aligned} f_{ix}^o &= -(1-\alpha_1)m(-a_x) \\ f_{jx}^o &= -\alpha_1 m(-a_x) \\ f_{kx}^o &= -(1-\alpha_2)m(-a_x) \\ f_{lx}^o &= -\alpha_2 m(-a_x) \end{aligned} \tag{24}$$

B) Y – direction (vertically)

The trolley's reaction on the structure in Y – direction is always existing. These are constant forces, which are considered in the static analysis. We simply offer the formula herein:

$$\begin{aligned} f_{iy}^o &= -(1-\alpha_1)mg \quad f_{jy}^o = -\alpha_1 mg \\ f_{ky}^o &= -(1-\alpha_2)mg \quad f_{ly}^o = -\alpha_2 mg \end{aligned} \tag{25}$$

3.2.2. The effect of the lifting weight and wire rope on the K and $f(t)$

(1) For the $f(t)$

A) In X – direction, reference to section 2) of 3.1, we have already known the lifting weight's reaction on the structure in this direction. It only needs to be distributed to the nodes of i, j, k and l based on following laws:

$$\begin{aligned} f_{ix}^w &= (1-\alpha_1)\cdot F_x/2 \quad f_{jx}^w = \alpha_1 \cdot F_x/2 \\ f_{kx}^w &= (1-\alpha_2)\cdot F_x/2 \quad f_{lx}^w = \alpha_1 \cdot F_x/2 \end{aligned} \tag{26}$$

B) Y – direction (vertically)

$$\begin{aligned} f_{iy}^w &= -(1-\alpha_1)\cdot F/2 \quad f_{jy}^w = -\alpha_1 \cdot F/2 \\ f_{ky}^w &= -(1-\alpha_2)\cdot F/2 \quad f_{ly}^w = -\alpha_2 \cdot F/2 \end{aligned} \tag{27}$$

Where $F = k_L(V_w' - V_w) > 0$ is unknown.

(2) For the stiffness matrix K and damping matrix C

From Eq. (27), we know that the force in Y – direction caused by the lifting weight is related to the displacements of the hanging point and the lifting weight's center of gravity (i.e. V_w' and V_w). In the actual calcu-

$$(d\Delta M(t))/dt = m \begin{array}{c} \begin{array}{ccccccccccccc} \ldots & u_i & v_i & \ldots & u_j & v_j & \ldots & u_k & v_k & \ldots & u_1 & v_1 & \ldots \end{array} \\ \begin{bmatrix} 0 &&&&&&&&&&&&&& \\ & \ddots &&&&&&&&&&&&& \\ && -\beta_1 &&&&&&&&&&&& \\ &&& -\beta_1 &&&&&&&&&&& \\ &&&& \ddots &&&&&&&&&& \\ &&&&& \beta_1 &&&&&&&&& \\ &&&&&& \beta_1 &&&&&&&& \\ &&&&&&& \ddots &&&&&&& \\ &&&&&&&& -\beta_2 &&&&&& \\ &&&&&&&&& -\beta_2 &&&&& \\ &&&&&&&&&& \ddots &&&& \\ &&&&&&&&&&& -\beta_2 &&& \\ &&&&&&&&&&&& -\beta_2 && \\ &&&&&&&&&&&&& \ddots & \\ &&&&&&&&&&&&&& 0 \end{bmatrix} \end{array} \tag{32}$$

$$\Delta K = 1/4k_L(t) = \begin{array}{c} \begin{array}{cccccccccc} \ldots & v_i & \ldots & v_j & \ldots & v_k & \ldots & v_l & \ldots \end{array} \\ \begin{bmatrix} 0\ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0\ldots & (1-\alpha_1)^2 & \ldots & (1-\alpha_1)\alpha_1 & \ldots & (1-\alpha_1)(1-\alpha_2) & \ldots & (1-\alpha_1)\alpha_2 & \ldots 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0\ldots & (1-\alpha_1)\alpha_1 & \ldots & \alpha_2^2 & \ldots & \alpha_1(1-\alpha_2) & \ldots & \alpha_1\alpha_2 & \ldots 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0\ldots & (1-\alpha_1)(1-\alpha_2) & \ldots & \alpha_1\alpha_2 & \ldots & (1-\alpha_2)^2 & \ldots & (1-\alpha_2)\alpha_2 & \ldots 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0\ldots & (1-\alpha_1)\alpha_2 & \ldots & \alpha_1\alpha_2 & \ldots & \alpha_2(1-\alpha_2) & \ldots & \alpha_2^2 & \ldots 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0\ldots & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & \ldots 0 \end{bmatrix} \end{array} \tag{34}$$

lation, first the V_w' in Eqs (7) and (21) is canceled by Eq. (20), and then substitute Eq. (27) into Eq. (21), and the items of node displacements in Eqs (7) and (21) are moved to their left side respectively. Finally we obtain the following global dynamic equation:

$$\underline{M}(t)\underline{\ddot{U}} + \underline{C}(t)\underline{\acute{U}} + \underline{K}(t)\underline{U} = \underline{f} \tag{28}$$

Where

$$\underline{M}(t) = \begin{bmatrix} M + \Delta M(t) & 0 \\ 0 & M_w \end{bmatrix} \tag{29}$$

$$\underline{C}(t) = \begin{bmatrix} C + (d\Delta M(r))/dt & 0 \\ 0 & C_w \end{bmatrix} \tag{31}$$

Where C is the structure damping matrix and C_w is the damping of the lifting weight.

Where $\beta_1 = (ds_1/dt)/l_1 = d\alpha_1/dt$, $\beta_2 = (ds_2/dt)/l_2 = d\alpha_2/dt$

$$\underline{K}(t) = \begin{bmatrix} K + \Delta K(t) & K_{UW} \\ K_{WU} & k_L(t) \end{bmatrix} \tag{33}$$

$$\underline{U} = [U^T, V_W]^T \tag{35}$$

Where

$$K_{WU} = K_{UW}^T$$

$$= 1/2k_L(t)[0,\ldots,\overset{v_i}{-(1-\alpha_1)},\ldots,\overset{v_j}{-\alpha_1},\ldots,\overset{v_k}{-(1-\alpha_2)},\ldots,\overset{v_1}{-\alpha_2},\cdots 0] \tag{33'}$$

We can get the dynamic response of the structure by solving the Eq. (28). The explicit central

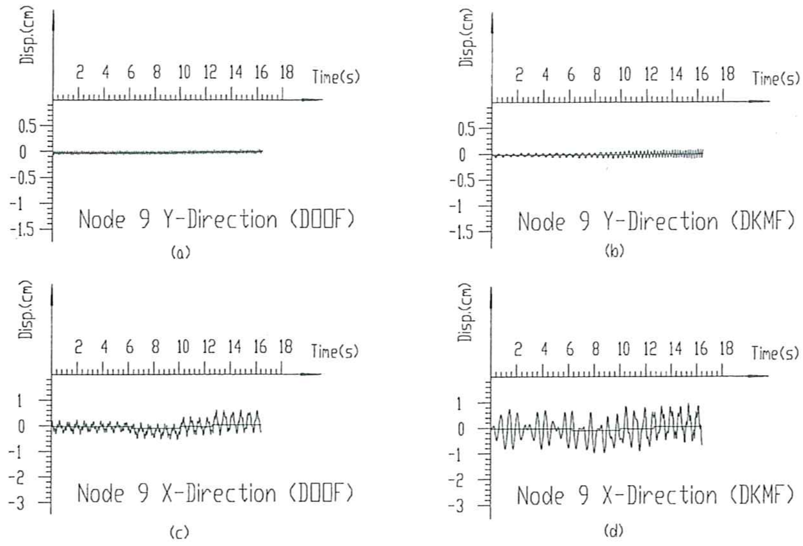

Fig. 6. Node 9 vibration.

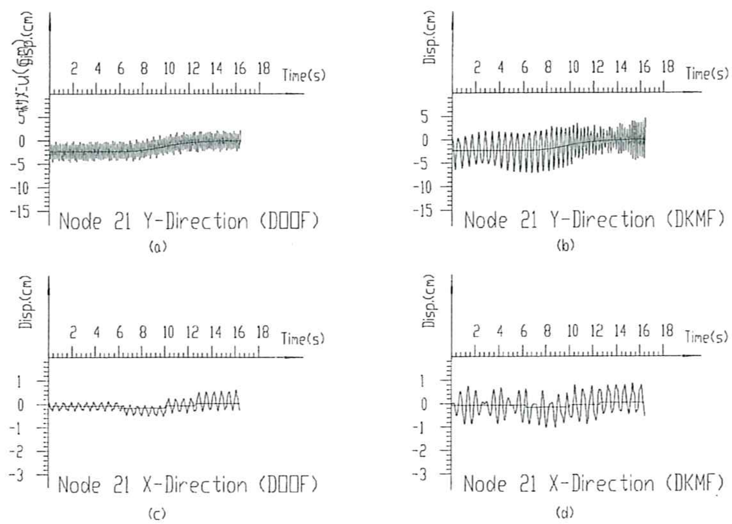

Fig. 7. Node 21 vibration.

difference method is used to solve the Eq. (28) in this paper. The unique condition for solving is that the time step length should be small enough, i.e. $\Delta t \leqslant \Delta t_{cr}(\Delta t_{cr} = T_n/\pi)$, Δt_{cr} is the critical step length and T_n is the minimal period of the structure system. In the following section, we give an example to show the actual application of this method.

4. Example

Figure 5 is the elevation sketch of a typical bucket ship unloader. Finding the maximal displacement of the key nodes from the phases of the hoisting of the lifting weight and the moving of the trolley to the phase of unloading (i.e. the dynamic response).

Given:

Mass of lifting weight = 20,000 kg, mass of bucket = 10,000 kg, mass of trolley = 34,000 kg, initial length of wire rope = 22 m, total metallic section area of wire rope = 0.0012 m^2, elastic modulus = 1011 N/m^2, hoist speed = 2.8 m/s, hoist acceleration = 0.78 m/s^2, trolley's speed Vx = 2.8 m/s, trolley's acceleration = 0.74 m/s^2. Sizes and geometrical properties for members in the structure. We neglect the influence of damping.

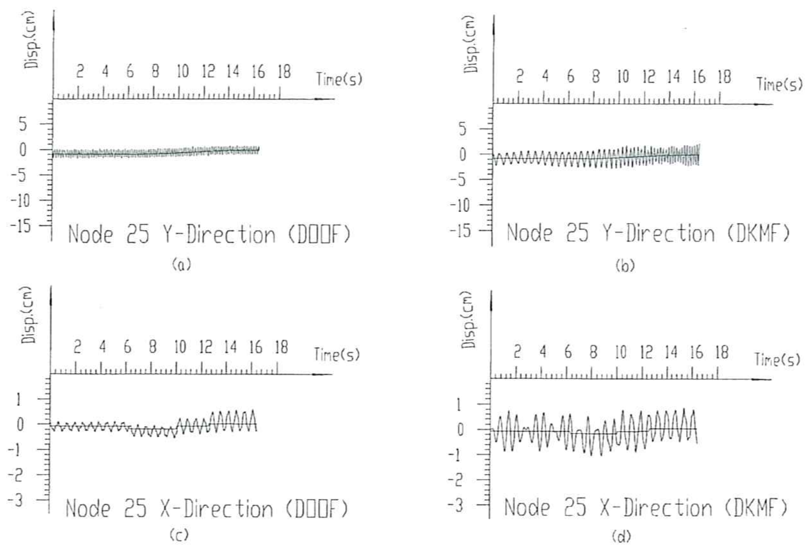

Fig. 8. Node 25 vibration.

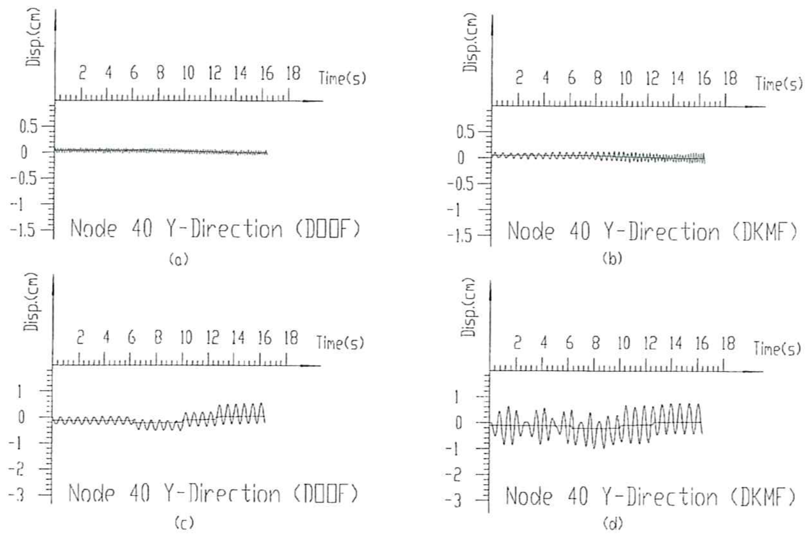

Fig. 9. Node 40 vibration.

We use Super SAP 91 and the central difference algorithm above mentioned respectively to analyze this example. The results are summarized in Table 1. (Table 1 only includes the maximal displacement of concerned nodes.)

As shown in Table 1, the analysis model in Super SAP 91 is in complete agreement with the one in DOOF (DOOF is a program using this method with no changing mass matrix M and stiffness matrix K). That is, at a certain moment we only take into account the influence of the weight's force in the structural dynamic response, neglecting both the variable stiffness of the wire rope and the vibration of the trolley and lifting weight. The analysis time is the hoisting phase (T0–T2). In Table 1 the results of Super SAP 91 and DOOF is comparatively close to each other when the conditions stay the same. The errors come from the different algorithm for solving the dynamic equation, because the method of Wilson-θ is used in Super SAP 91, we choose the explicit center difference method. In DKMF(1) and DKMF(2) (DKMF is a program using this method with changing mass matrix M and stiffness matrix K), the variations of mass matrix and stiffness matrix are taken into account. The analysis time interval in DKMF(1) is

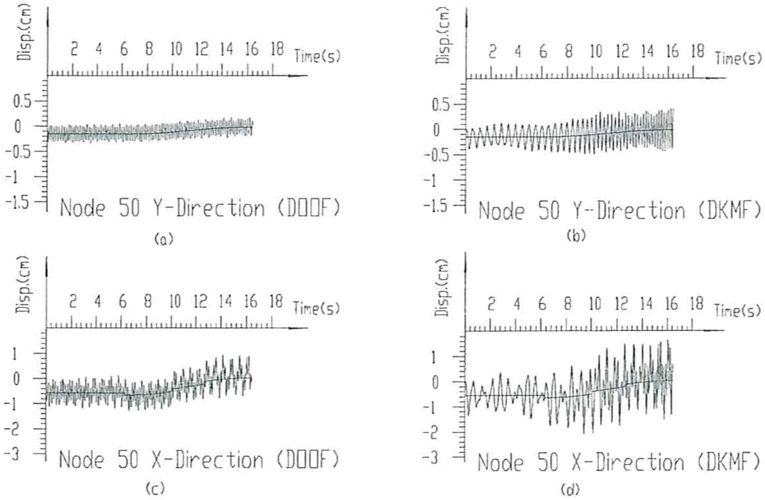

Fig. 10. Node 50 vibration.

Table 1
Displacement (m) comparison of key nodes

NODE		Super SAP91 $T = 0 - 6.24$ s	DOOF $T = 0 - 6.24$ s	DKMF(1) $T = 0 - 6.24$ s	DKMF(2) T=0-16.5s	
		<F> change	<F> change	[M],[K],<F> change	[M],[K],<F> change	
9	X	3.20E-3	3.19E-3	8.15E-3	9.75-3	$(T = 8.8)$
	Y	6.20E-4	6.10E-4	6.90E-4	9.17E-4	$(T = 15.72)$
11	X	3.16-3	3.20E-3	8.24E-3	9.86E-3	$(T = 8.8)$
	Y	3.97E-4	3.65E-4	4.10E-4	5.00E-4	$(T = 9.4)$
21	X	2.83E-3	2.97E-3	8.84E-3	1.04E-2	$(T = 8.8)$
	Y	4.76E-2	4.67E-2	6.94E-2	7.16E-2	$(T = 6.72)$
25	X	2.83E-3	2.97E-3	8.80E-3	1.04E-E2	$(T = 8.0)$
	Y	1.81E-2	1.76E-2	2.46E-2	2.74E-2	$(T = 9.4)$
29	X	2.81E-3	2.96E-3	8.80E-3	1.03E-2	$(T = 8.0)$
	Y	7.68E-3	4.95E-3	4.40E-3	9.45E-3	$(T = 12.68)$
33	X	2.79E-3	2.97E-3	8.70E-3	1.01E-2	$(T = 8.0)$
	Y	1.64E-3	1.62E-3	1.83E-3	2.43E-3	$(T = 15.72)$
40	X	2.80E-3	3.00E-3	8.8E-3	1.00E-2	$(T = 8.0)$
	Y	1.00E-3	9.30E-4	1.10E-3	1.27E-3	$(T = 9.4)$
50	X	1.17E-2	1.16E-2	1.47E-2	2.12E-2	$(T = 9.4)$
	Y	3.43E-3	3.40E-3	4.34E-3	5.06E-3	$(T = 9.4)$

T0–T2 (the hoisting phase). The analysis time interval in DKMF(2) is T0–T8 (the whole process, from ascending to travelling and unloading). One can clearly find out that the difference of the maximal displacement between considering the variations of M, K and not from Table 1. We can see from the results, the maximal displacement occurs after the moment of $T > 6.24$ s (the hoisting phase). These results are also distinctly different from those of the traditional dynamic response analysis in which the maximal displacement occurred in the hoisting. Also the maximal values between the two methods are also distinctly different. For the sake of intuitive observation, we draw the variation of displacement in x and y directions of the concerned nodes in T0–T8 shown from Fig. 6 to Fig. 10. There are significant difference between considering the changes of (M, K) and not.

5. Conclusions

1) By comparing the analysis results of this method with that of Super SAP 91, we find:
   (1) This method is correct and reliable.
   (2) The distinct difference of the results shows the necessity of this method.
2) The traditional design only takes into account the ascent phase. This phase is thought of the most